

## SUCCESSIVE - DIFFERENTIAL

1. If  $y = f(x)$  then  $f^{(n)}(x)$  is denoted by  $\frac{d^n y}{dx^n}$  or  $D^n y$  or  $y^{(n)}$  or  $y_n$ .
2.  $\frac{d^n y}{dx^n} = \frac{d}{dx} \left\{ \frac{d^{n-1} y}{dx^{n-1}} \right\}$ .
3. If  $y = (ax + b)^m$ , then  $y_n = m(m-1)(m-2) \dots (m-n+1)(ax + b)^{m-n} a^n$ .
4. If  $f(x) = (ax + b)^m$ ,  $m \in \mathbb{Z}$ ,  $m > 0$ ,  $n \in \mathbb{N}$  then
  - (i)  $m < n \Rightarrow f^{(n)}(x) = 0$
  - (ii)  $m = n \Rightarrow f^{(n)}(x) = n! a^n$
  - (iii)  $m > n \Rightarrow f^{(n)}(x) = \frac{m!}{(m-n)!} (ax + b)^{m-n} a^n$ .
5. If  $f(x)$  is a polynomial function of degree less than  $n$  where  $n \in \mathbb{N}$ , the  $f^{(n)}(x) = 0$ .
6. If  $y = \frac{1}{ax + b}$ , then  $y_n = \frac{(-1)^n n! a^n}{(ax + b)^{n+1}}$
7. If  $y = \log |ax + b|$  then  $y_n = \frac{(-1)^{n-1} (n-1)! a^n}{(ax + b)^n}$ .
8. If  $y = \sin(ax + b)$ , then  $y_n = a^n \sin(ax + b + n\pi/2)$ .
9. If  $y = \cos(ax + b)$ , then  $y_n = a^n \cos(ax + b + n\pi/2)$ .
10. If  $y = e^{ax+b}$ , then  $y_n = a^n e^{ax+b}$
11. If  $y = a^{bx+c}$ , then  $y_n = b^n a^{bx+c} (\log a)^n$ .
12. If  $y = e^{ax} \sin(bx + c)$ , then  $y_n = r^n e^{ax} \sin(bx+c+n\theta)$  where  $a = r \cos \theta$ ,  $b = r \sin \theta$ . Now  $r = \sqrt{a^2 + b^2}$ ,  $\theta = \tan^{-1}(b/a)$ .
13. If  $y = e^{ax} \cos(bx + c)$ , then  $y_n = r^n e^{ax} \cos(bx+c+n\theta)$  where  $a = r \cos \theta$ ,  $b = r \sin \theta$ . Now  $r = \sqrt{a^2 + b^2}$ ,  $\theta = \tan^{-1}(b/a)$ .
14. **Leibnitz's theorem** : If  $f, g$  are two functions in  $x$  having  $n^{\text{th}}$  derivatives then
 
$$(fg)^{(n)}(x) = {}^n C_0 f^{(n)}(x)g(x) + {}^n C_1 f^{(n-1)}(x)g'(x) +$$

$${}^n C_2 f^{(n-2)}(x)g^{(2)}(x) + \dots + {}^n C_r f^{(n-r)}(x)g^{(r)}(x) + \dots + {}^n C_n f(x)g^{(n)}(x).$$
15. If  $u, v$  are two functions in  $x$ , then  $(u, v)_n =$ 

$${}^n C_0 u_n v + {}^n C_1 u_{n-1} v_1 + {}^n C_2 u_{n-2} v_2 + \dots + {}^n C_n u v_n =$$

$${}^n C_0 u_1 v_{n-1} + {}^n C_1 u_1 v_{n-1} + {}^n C_2 u_2 v_{n-2} + \dots + {}^n C_n u_n v.$$